

Name (IN CAPITALS): **Version #1**

Instructor: Dora The Explorer

Math 10560 Exam 1

Sept. 22, 2026.

- The Honor Code is in effect for this examination. All work is to be your own.
- Please put all cellphones, smartwatches and electronic devices out of reach.
- Calculators are **not** allowed.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-16.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. ☐ (•) (b) (c) (d) (e)

2. ☐ (•) (b) (c) (d) (e)

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7. (•) (b) (c) (d) (e)

8. (•) (b) (c) (d) (e)

9. (•) (b) (c) (d) (e)

10. (•) (b) (c) (d) (e)

11. (•) (b) (c) (d) (e)

12. (•) (b) (c) (d) (e)

Please do NOT write in this box.

Multiple Choice _____

13. _____

14. _____

15. _____

16. _____

Total _____

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1. (a) (b) (c) (d) (e)

2 (a) (b) (c) (d) (e)

3. (a) (b) (c) (d) (e)

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10. (a) (b) (c) (d) (e)

11. (a) (b) (c) (d) (e)

12. (a) (b) (c) (d) (e)

Please do NOT write in this box.

Multiple Choice _____

13. _____

14. _____

15. _____

16. _____

Total _____

2.

Initials: _____

Multiple Choice

1.(7pts) The function $f(x) = 3x + 2\ln(x)$ is one-to-one. Find $(f^{-1})'(3)$.

(a) $\frac{1}{5}$

(b) $\frac{1}{3}$

(c) 5

(d) $\frac{3}{11}$

(e) $\frac{11}{3}$

Solution. Observe that

$$f(1) = 3(1) + 2\ln(1) = 3.$$

Then we have $f^{-1}(3) = 1$. Using

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))},$$

and

$$f'(x) = 3 + \frac{2}{x},$$

we obtain

$$(f^{-1})'(3) = \frac{1}{f'(1)} = \frac{1}{3+2} = \boxed{\frac{1}{5}}.$$

3.

Initials: _____

2.(7pts) Evaluate the integral

$$\int_0^1 \frac{e^x + 1}{e^x + x + 2} dx$$

(a) $\ln(e + 3) - \ln(3)$

(b) -1

(c) $\frac{1}{(e + 3)^2} - \frac{1}{4}$

(d) $\ln(e + 3) - \ln(2)$

(e) $\frac{2}{(e + 3)^2} - \frac{2}{9}$

Solution. Let

$$u = e^x + x + 2.$$

Then

$$du = (e^x + 1) dx.$$

Then we compute the new boundary values, and we have

$$x = 0 \implies u = 3, \quad x = 1 \implies u = e + 3.$$

Then we have

$$\int_0^1 \frac{e^x + 1}{e^x + x + 2} dx = \int_3^{e+3} \frac{1}{u} du = [\ln |u|]_3^{e+3}.$$

Plugging in the boundary values, we obtain

$$\int_0^1 \frac{e^x + 1}{e^x + x + 2} dx = \boxed{\ln(e + 3) - \ln(3)}.$$

4.

Initials: _____

3.(7pts) Find $\frac{dy}{dx}$ if

$$y = (2 + \sin x)^x.$$

(a) $(2 + \sin x)^x \left[\ln(2 + \sin x) + \frac{x \cos x}{2 + \sin x} \right]$

(b) $\left[\ln(2 + \sin x) - \frac{x \cos x}{2 + \sin x} \right]$

(c) $(2 + \sin x)^x \left[\ln(2 + \sin x) + \frac{x}{2 + \sin x} \right]$

(d) $(2 + \sin x)^x \left[\frac{\cos x}{2 + \sin x} \right]$

(e) $x(2 + \sin x)^{x-1}$

Solution. We use logarithmic differentiation. Taking the natural logarithm of both sides, we have

$$\ln y = x \ln(2 + \sin x).$$

Differentiating both sides and using the product rule, we have

$$\frac{y'}{y} = \ln(2 + \sin x) + x \left(\frac{\cos x}{2 + \sin x} \right).$$

Multiplying by $y = (2 + \sin x)^x$, we obtain

$$\frac{dy}{dx} = (2 + \sin x)^x \left[\ln(2 + \sin x) + \frac{x \cos x}{2 + \sin x} \right].$$

5.

Initials: _____

4.(7pts) Find $\frac{dy}{dx}$ if

$$y = 2^{\cos^2 x}$$

(a) $-2^{1+\cos^2 x} \ln(2) \cos x \sin x$

(b) $(\cos^2 x) 2^{(\cos^2 x)-1}$

(c) $2^{1+\cos^2 x} \ln(2) \cos x$

(d) $-2^{\cos^2 x} \ln(2) \sin^2 x$

(e) $\frac{2^{\cos^2 x}}{\ln(2)}$

Solution. By the chain rule, we have

$$\frac{dy}{dx} = 2^{\cos^2 x} \ln(2) \cdot 2 \cos x \cdot (-\sin x) = \boxed{-2^{1+\cos^2 x} \ln(2) \cos x \sin x}.$$

- 5.(7pts) A sample of radioactive material (which decays exponentially) decays to $1/4$ of its original amount in one day. How long (in days) would it take to decay to $1/20$ of its original amount?

(a) $\frac{\ln(20)}{\ln(4)}$ (b) $-\frac{\ln(20)}{\ln(4)}$ (c) $-4 \ln(20)$ (d) $\ln(5)$ (e) $4 \ln(20)$

Solution. Because the material retains $1/4$ of its amount each day, the formula for an exponential-decay model is

$$A(t) = A_0 \left(\frac{1}{4} \right)^t,$$

where A_0 is the original amount and t is measured in days. Next, notice the question asks for the t value when

$$A(t) = \frac{A_0}{20}.$$

Plugging in the formula, we have

$$\frac{A_0}{20} = A_0 \left(\frac{1}{4} \right)^t.$$

Canceling A_0 gives

$$\frac{1}{20} = \left(\frac{1}{4} \right)^t.$$

Taking natural logarithms on both sides, we have

$$\ln \left(\frac{1}{20} \right) = t \ln \left(\frac{1}{4} \right).$$

Therefore, we obtain

$$t = \frac{\ln(1/20)}{\ln(1/4)} = \boxed{\frac{\ln(20)}{\ln(4)}}.$$

7.

Initials: _____

6.(7pts) Calculate the following indefinite integral.

$$\int \frac{x^3}{\sqrt{1-x^8}} dx .$$

(a) $\frac{\sin^{-1}(x^4)}{4} + C$

(b) $\sin^{-1}(x^4) + C$

(c) $\frac{\ln(\sqrt{1-x^8})}{4} + C$

(d) $\frac{\tan^{-1}(x^4)}{4} + C$

(e) $\tan^{-1}(x^4) + C$

Solution. Let

$$u = x^4.$$

Then

$$du = 4x^3 dx.$$

Then we have

$$\int \frac{x^3}{\sqrt{1-x^8}} dx = \frac{1}{4} \int \frac{1}{\sqrt{1-u^2}} du = \frac{1}{4} \sin^{-1}(u) + C.$$

Substituting back, we obtain

$$\int \frac{x^3}{\sqrt{1-x^8}} dx = \boxed{\frac{\sin^{-1}(x^4)}{4} + C}.$$

8.

Initials: _____

7.(7pts) Find $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$.

(a) $-\frac{1}{6}$

(b) $-\frac{1}{3}$

(c) 0

(d) $\frac{1}{2}$

(e) Does not exist

Solution. First notice direct substitution gives the indeterminate form $\frac{0}{0}$. Therefore, we apply L'Hopital's Rule, and we have

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} = \lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2}.$$

Another direct substitution still gives the indeterminate form $\frac{0}{0}$. Therefore, we apply L'Hopital's Rule a second time, and we have

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2} = \lim_{x \rightarrow 0} \frac{-\sin x}{6x}.$$

Direct substitution again gives the indeterminate form $\frac{0}{0}$. Therefore, we apply L'Hopital's Rule a third time, and we have

$$\lim_{x \rightarrow 0} \frac{-\sin x}{6x} = \lim_{x \rightarrow 0} \frac{-\cos x}{6}.$$

One last direct substitution gives

$$\lim_{x \rightarrow 0} \frac{-\cos x}{6} = -\frac{\cos(0)}{6} = \boxed{-\frac{1}{6}}.$$

9.

Initials: _____

8.(7pts) Find

$$\int_1^2 x^3 (\ln x) \, dx.$$

(a) $4 \ln(2) - \frac{15}{16}$

(b) $4 \ln(2) - 1$

(c) $16 \ln(2) - \frac{15}{4}$

(d) $\frac{7}{4}$

(e) $4 \ln(2)$

Solution. Using integration by parts with

$$u = \ln x, \quad dv = x^3 \, dx,$$

Then

$$du = \frac{1}{x} \, dx, \quad v = \frac{x^4}{4}.$$

Then we have

$$\begin{aligned} \int_1^2 x^3 \ln x \, dx &= \left[\frac{x^4}{4} \ln x \right]_1^2 - \frac{1}{4} \int_1^2 x^3 \, dx \\ &= 4 \ln 2 - \frac{1}{16} [x^4]_1^2 \\ &= 4 \ln 2 - \frac{16 - 1}{16} \\ &= \boxed{4 \ln 2 - \frac{15}{16}}. \end{aligned}$$

10.

Initials: _____

9.(7pts) Compute $\int_0^{\pi/2} \cos(2x) \cos(x) dx$
(Hint: The formula sheet may help.)

(a) $\frac{1}{3}$

(b) 0

(c) $-\frac{2}{3}$

(d) $-\frac{1}{3}$

(e) $\frac{2}{3}$

Solution. Using the product-to-sum identity in the formula sheet, we have

$$\cos(2x) \cos x = \frac{1}{2}(\cos x + \cos(3x)).$$

Therefore,

$$\begin{aligned} \int_0^{\pi/2} \cos(2x) \cos x dx &= \frac{1}{2} \int_0^{\pi/2} (\cos x + \cos(3x)) dx \\ &= \frac{1}{2} \left[\sin x + \frac{\sin(3x)}{3} \right]_0^{\pi/2} \\ &= \frac{1}{2} \left(1 - \frac{1}{3} \right) \\ &= \boxed{\frac{1}{3}}. \end{aligned}$$

10.(7pts) Find $\int_0^{\pi/4} \tan^3 x \sec^4 x \, dx$.

(a) $\frac{5}{12}$

(b) $\frac{1}{12}$

(c) $\frac{1}{6}$

(d) $-\frac{1}{6}$

(e) $\frac{1}{10}$

Solution. Separating one factor of $\sec^2 x$ and using $\sec^2 x = 1 + \tan^2 x$, we have

$$\int_0^{\pi/4} \tan^3 x \sec^4 x \, dx = \int_0^{\pi/4} \tan^3 x (1 + \tan^2 x) \sec^2 x \, dx.$$

Let

$$u = \tan x.$$

Then

$$du = \sec^2 x \, dx.$$

Then the new boundary values become

$$x = 0 \implies u = \tan(0) = 0, \quad x = \frac{\pi}{4} \implies u = \tan\left(\frac{\pi}{4}\right) = 1.$$

Therefore, we have

$$\begin{aligned} \int_0^{\pi/4} \tan^3 x \sec^4 x \, dx &= \int_0^1 u^3 (1 + u^2) \, du \\ &= \left[\frac{u^4}{4} + \frac{u^6}{6} \right]_0^1 \\ &= \frac{1}{4} + \frac{1}{6} \\ &= \boxed{\frac{5}{12}}. \end{aligned}$$

11.(7pts) The decomposition of $\frac{x-3}{(x-1)(x^2-1)}$ into partial fractions is

(a) $\frac{1}{x-1} - \frac{1}{(x-1)^2} - \frac{1}{x+1}$

(b) $-\frac{1}{x-1} + \frac{1}{(x-1)^2} + \frac{1}{x+1}$

(c) $\frac{1}{x-1} + \frac{1}{(x-1)^2} + \frac{2}{x+1}$

(d) $-\frac{2}{x-1} + \frac{1}{x+1}$

(e) $-\frac{1}{(x-1)^2} + \frac{1}{x+1}$

Solution. First notice we can factor the denominator as

$$(x-1)(x^2-1) = (x-1)^2(x+1).$$

Therefore, we can write

$$\frac{x-3}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}.$$

Multiplying both sides by $(x-1)^2(x+1)$ gives

$$x-3 = A(x-1)(x+1) + B(x+1) + C(x-1)^2.$$

Setting $x = 1$, we have $B = -1$. Setting $x = -1$, we have $C = -1$. Finally, setting $x = 0$ and plugging in $B = -1$ and $C = -1$, we have $A = 1$. Therefore, we have

$$\frac{x-3}{(x-1)(x^2-1)} = \boxed{\frac{1}{x-1} - \frac{1}{(x-1)^2} - \frac{1}{x+1}}.$$

12.(7pts) Evaluate

$$\int \frac{2}{x^2 + x - 2} dx.$$

(a) $\frac{2}{3} \ln \left(\left| \frac{x-1}{x+2} \right| \right) + C$

(b) $\frac{2}{3} \ln \left(\left| \frac{x+2}{x-1} \right| \right) + C$

(c) $2 \ln \left(\left| \frac{x-1}{x+2} \right| \right) + C$

(d) $2 \ln \left(\left| \frac{x+1}{x-2} \right| \right) + C$

(e) $2 \ln \left(\left| \frac{x+2}{x-1} \right| \right) + C$

Solution. First notice we can factor the denominator as

$$x^2 + x - 2 = (x-1)(x+2).$$

Therefore, we can write

$$\frac{2}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

Multiplying both sides by $(x-1)(x+2)$, we have

$$2 = A(x+2) + B(x-1).$$

Setting $x = 1$, we have $A = 2/3$, and setting $x = -2$, we have $B = -2/3$. Therefore, we have

$$\begin{aligned} \int \frac{2}{x^2 + x - 2} dx &= \frac{2}{3} \int \frac{1}{x-1} dx - \frac{2}{3} \int \frac{1}{x+2} dx \\ &= \frac{2}{3} \ln |x-1| - \frac{2}{3} \ln |x+2| + C \\ &= \boxed{\frac{2}{3} \ln \left| \frac{x-1}{x+2} \right| + C}. \end{aligned}$$

Partial Credit

For full credit on partial credit problems, make sure you justify your answers.

13.(12pts) Calculate the integral

$$\int \frac{dx}{\sqrt{x^2 + 25}}$$

Note: The formula sheet will help you with this problem.

Write your answer in terms of the original variable x and (if needed) replace all composite trigonometric functions (such as $\cos(\sin^{-1}(x/n))$ etc...) by algebraic combinations of x .

Solution. This is a trig-substitution problem. First, we identify that the integral contains an expression of the form $\sqrt{x^2 + a^2}$ and recognize that we need to use the substitution

$$x = a \tan \theta,$$

where $a = 5$. Then we have

$$dx = 5 \sec^2 \theta d\theta.$$

Then our integral becomes

$$\int \frac{dx}{\sqrt{x^2 + 25}} = \int \frac{5 \sec^2 \theta}{\sqrt{(5 \tan \theta)^2 + 25}} d\theta = \int \frac{5 \sec^2 \theta}{5 \sqrt{\tan^2 \theta + 1}} d\theta = \int \frac{\sec^2 \theta}{\sqrt{\sec^2 \theta}} d\theta = \int \sec \theta d\theta.$$

Notice the technicality that we need to choose $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, so that $\sec \theta > 0$ and therefore $\sqrt{\sec^2 \theta} = \sec \theta$. Next, recall the formula

$$\int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C.$$

Then we have

$$\int \frac{dx}{\sqrt{x^2 + 25}} = \ln |\sec \theta + \tan \theta| + C.$$

Then because

$$\tan \theta = \frac{x}{5},$$

we use

$$1 + \tan^2 \theta = \sec^2 \theta$$

to obtain

$$\sec \theta = \sqrt{1 + \frac{x^2}{25}} = \frac{\sqrt{x^2 + 25}}{5}.$$

Substituting back in terms of x , we have

$$\int \frac{dx}{\sqrt{x^2 + 25}} = \ln \left| \frac{\sqrt{x^2 + 25}}{5} + \frac{x}{5} \right| + C = \ln \left| \frac{x + \sqrt{x^2 + 25}}{5} \right| + C = \ln |x + \sqrt{x^2 + 25}| + C,$$

where the constant $-\ln(5)$ has been absorbed into C in the last equality. Hence, our final answer is

$$\int \frac{dx}{\sqrt{x^2 + 25}} = \boxed{\ln |x + \sqrt{x^2 + 25}| + C}.$$

14.(12pts) Evaluate the limit

$$\lim_{x \rightarrow 0} (\cos(3x))^{1/x^2}.$$

Solution. First, we recognize that direct substitution of the value $x = 0$ gives the indeterminate form 1^∞ . We then define

$$L = \lim_{x \rightarrow 0} (\cos(3x))^{1/x^2}.$$

and we take the natural logarithm on both sides. It follows that

$$\ln(L) = \lim_{x \rightarrow 0} \frac{\ln(\cos(3x))}{x^2}.$$

Then notice that a direct substitution of the value $x = 0$ on the right hand side gives the indeterminate form $0/0$. Therefore, we apply L'Hopital's Rule. Then we have

$$\ln(L) = \lim_{x \rightarrow 0} \frac{-3 \tan(3x)}{2x}.$$

Again, direct substitution of the value $x = 0$ on the right hand side still gives the indeterminate form $0/0$. Therefore, we apply L'Hopital's Rule again. Then we have

$$\begin{aligned} \ln(L) &= \lim_{x \rightarrow 0} \frac{-9 \sec^2(3x)}{2} \\ &= -\frac{9 \sec^2(0)}{2} \\ &= -\frac{9}{2}. \end{aligned}$$

Finally, exponentiating both sides gives

$$L = e^{\ln(L)} = e^{-9/2}.$$

We then obtain the final answer

$$\lim_{x \rightarrow 0} (\cos(3x))^{1/x^2} = \boxed{e^{-9/2}}.$$

True-False.

- 15.(4pts) Please circle "TRUE" if you think the statement is true, and circle "FALSE" if you think the statement is False.

(a)(1 pt. No Partial credit) If the size of a population of bacteria after t days is given by $P(t) = P(0)e^{0.5t}$ where $P(0)$ is the size of the population at time $t = 0$, then the time that it takes for the population size to double (i.e. to reach $2P(0)$ in size) does not depend on the initial population size ($P(0)$).

TRUE FALSE

(b)(1 pt. No Partial credit) When evaluating the integral $\int \sqrt{x^2 - 9} \, dx$, the substitution $x = 3 \sin \theta$ gives $\int \sqrt{x^2 - 9} \, dx = 9 \int \cos^2 \theta \, d\theta$.

TRUE FALSE

(c)(1 pt. No Partial credit) $\tan^{-1}(\tan x) = x$ for all values of x in the domain of $\tan x$.

TRUE FALSE

(d)(1 pt. No Partial credit) $\lim_{x \rightarrow \infty} 2^{1/x} = 1$.

TRUE FALSE

Solution.

(a). **TRUE.** Let $2P(0) = P(t) = P(0)e^{0.5t}$. Dividing by $P(0)$, we have $2 = e^{0.5t}$. Taking natural logarithms gives $\ln 2 = 0.5t$, and hence $t = 2 \ln 2$. Thus, the doubling time does not depend on $P(0)$.

(b). **FALSE.** The expression $\sqrt{x^2 - 9}$ has the form $\sqrt{x^2 - a^2}$, so the appropriate trigonometric substitution is $x = 3 \sec \theta$. Indeed, using $x = 3 \sin \theta$ gives $\sqrt{x^2 - 9} = 3\sqrt{\sin^2 \theta - 1}$, which does not simplify to $3 \cos \theta$. Moreover, since $dx = 3 \cos \theta \, d\theta$, the original integral becomes $9 \int \cos \theta \sqrt{\sin^2 \theta - 1} \, d\theta$, not $9 \int \cos^2 \theta \, d\theta$.

(c). **FALSE.** The range of $\tan^{-1} x$ is $(-\frac{\pi}{2}, \frac{\pi}{2})$. Therefore, $\tan^{-1}(\tan x) = x$ only when $-\frac{\pi}{2} < x < \frac{\pi}{2}$, while the domain of $\tan x$ is all real numbers except $x = \frac{\pi}{2} + k\pi$, where k is an integer. For instance,

$$\tan^{-1}(\tan \pi) = \tan^{-1}(0) = 0 \neq \pi.$$

(d). **TRUE.** Because the exponential function $f(u) = 2^u$ is continuous and $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$, direct substitution works for this limit. Therefore, we have $\lim_{x \rightarrow \infty} 2^{1/x} = 2^0 = 1$.

17.

Initials: _____

16.(2pts) You will be awarded these two points if you write your name in CAPITALS on the front page and you mark your answers on the front page with an X through your answer choice like so: ~~A~~ (not an O around your answer choice) .

The following is the list of useful trigonometric formulas:

Note: $\sin^{-1} x$ and $\arcsin(x)$ are different names for the same function and $\tan^{-1} x$ and $\arctan(x)$ are different names for the same function.

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}(\sin(x - y) + \sin(x + y))$$

$$\sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x - y) + \cos(x + y))$$

$$\int \sec \theta = \ln |\sec \theta + \tan \theta| + C$$

$$\int \csc \theta = \ln |\csc \theta - \cot \theta| + C$$

$$\csc \theta = \frac{1}{\sin \theta}, \quad \cot \theta = \frac{1}{\tan \theta}$$

19.

Initials: _____

ROUGH WORK